Sample Level 2 Editing

A Lattice Model for Option Pricing Under GARCH-Jump Processes

ABSTRACT

This study posits a need for an innovative discrete-time lattice model ~~This study integrates the GARCH option pricing tree of Ritchken and Trevor (1999) and the jump-diffusion option pricing tree of Amin (1993) to obtain a new discrete-time lattice model to value options~~ in circumstances where ~~when~~ the underlying processing of value options follows a mixture of a GARCH and a discontinuous jump process. In this regard, Ritchken and Trevor's (1999) Garch option pricing tree demonstrably proves complementary to the function of Amin's (1993) jump diffusion pricing tree. Moreover, this ~~The assumption of a~~ GARCH-jump model provides a better description for the underlying process given its consisten~~t~~cy with the empirical results~~,~~; ~~in which the~~ discontinuities in the sample path of financial assets are found even after allowing for conditional heteroskedasticity. The lattice model can also adapt to ~~the~~ Duan, Ritchken, and Sun's (2006) GARCH-jump model in pricing American-~~–~~style options~~,~~. ~~and t~~Furthermore, the Amin (1993), ~~the~~ Ritchken and Trevor (1999), ~~the~~ Cox, Ross, and Rubinstein (1979), ~~and~~ along with ~~the~~ Kamrad and Ritchken's (1991) models, are ~~nested to~~ interwoven with our generalized lattice model. ~~Numerical results of our model~~ Our numerical results are consistent with ~~the results of~~ the Monte Carlo simulations for pricing European options under the GARCH-jump process. ~~For~~ With respect to American options, our ~~results illustrate~~ data indicates that the ~~early exercise~~early-exercise premium decreases with the increases of the jump intensity, or the mean of the jump magnitude. In addition, ~~the~~ results for the implied volatility show that the generalized model can capture the volatility smile ~~as well as~~ and the term structure of volatility effects observed in options markets.

Keywords: GARCH option model, jump-diffusion option model, GARCH-jump option model, lattice model

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I. INTRODUCTION

The stochastic ~~process~~ nature of stock prices is ~~a crucial issue~~ of pivotal importance for option valuations. Conventional assumptions have treated this diffusion process as continuous~~Stock prices are conventionally assumed to follow a diffusion process with continuous path~~. For example, t~~T~~he seminal Black-Scholes (1973) option pricing model assumes that stock prices follow the Geometric Brownian Motion (GBM), in which the conditional distribution of stock (continuously compounded) returns is normal. Such conclusions are qualified, though, by ~~However,~~ empirical evidence, which indicates that biases, such as the high level of kurtosis and non-zero skewness found in stock return distributions, ~~are embedded in this assumption~~cannot be discounted as influencing factors. ~~As a result,~~ Consequently, observable market options prices~~ observed in the market~~, relative to the Black-Scholes model, ~~generally exhibit~~ tend to manifest in ~~certain~~ patterns, such as the volatility smile and the term structure of volatility effects.

~~One of an important and~~ A widely adopted remedy toward this issue ~~is~~ holds that the conditional variance in the underlying process is stochastic or time varying, where the asset (unconditional) returns distribution displays ~~in~~ skewness ~~as well as~~ in addition to leptokurtosis. ~~Along~~ In this vein, the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) ~~type of~~ processes are discrete-time models able to capture ~~the phenomenon of~~ stochastic changes in volatility~~ over time~~. Under the GARCH framework, Duan (1995) applies the concept of locally risk-neutral valuation relationship to develop an ~~option pricing~~option-pricing model, thereby ~~and~~ solv~~es~~ing the option price by large sample simulation methods.

The complication of option pricing under the GARCH process is largely attributable ~~mainly due~~ to its path-dependent characteristics~~,~~. ~~which cause~~ When pricing American-style options, t~~t~~he resulting non-recombined effect leads to both an exponential increase in the nodes of the lattice model and their associated temporal steps~~thus the number of nodes in the lattice model grows exponentially along with the increase of time steps when pricing American-style options~~. ~~Realizing~~ Cognizant of this complexity, Ritchken and Trevor (1999) developed an efficient lattice algorithm to price, in particular ~~American style~~American-style options, under the discrete-time GARCH process. ~~Further,~~ Cakici and Tapyan (2000) ~~provide a more efficient method by modifying the~~ modified Ritchken and Trevor's (1999) model, ~~and they point out~~ thus highlighting a faster ~~that the~~ convergence rate in combination with a more efficient ~~is faster and the~~ computational time ~~is more efficient under their modified method~~.

~~Although~~ Attractive as the GARCH processes may be~~are attractive, however~~, they are ~~not~~ unable to account for ~~describe the occasional and large discrete~~ any changes embedded in asset price behaviors. An alternative model, ~~that can~~ capable of captur~~e~~ing the fat-tail effect and skewness in the return distribution caused by the occasionally large movements in asset price, is the jump-diffusion process. Merton (1976) proposed a jump-diffusion option pricing model, where a Poisson jump process is added to the Geometric Brownian Motion for the underlying asset price. The jump-diffusion model is based on the assumption that stock returns are generated by a mixture of processes, ~~which~~ includ~~es~~ing ~~small,~~ continuous, incremental fluctuations to prices ~~movements~~ generated by a Wiener process, and large, infrequent price jumps pertaining to nonsystematic risk~~,~~. ~~generated by a Poisson process~~A Poisson process generates the latter. ~~In particular, t~~The jump process is used to specifically describe ~~the~~ abnormal price variations ~~vibrations in price~~ due to ~~the arrival of important~~ new information pertaining to ~~about~~ the financial market that ~~turn out~~ has more than a marginal effect on price. The model can provide an explanation for empirically observ~~ed~~able distributions of stock price changes that exhibit skewness, leptokurtosis, and fatter tails in both sides, relative to the lognormal assumption in the Black-Scholes model.

Besides Merton (1976), Cox and Ross (1976)~~,~~ and Ahn and Thompson (1988)

~~also~~ derive option pricing models by assuming discontinuous jump processes for the price of underlying assets. Their models are ~~similar~~ comparable to Merton's (1976), insofar as they ~~that can~~ provide a reasonable explanation for underestimation of ~~the observations that~~ prices of out-of-money and close to maturity options~~ are underestimated~~. They also facilitate comparable evaluations of ~~and~~ the effectiveness of short-term hedging strategies based on the dynamic portfolio adjustment, suggesting they are ~~is~~ overestimated when associated with conventional pricing models.~~.~~

~~The~~ Merton's (1976) model is a quasi-closed form model for pricing European options. In order t~~T~~o price a wider range of options, Amin (1993) develop~~s~~ed a discrete-time lattice model, which although derived ~~is rigorously extended~~ from the Cox, Ross, and Rubinstein~~'s~~ (1979) binomial tree, also marks an extension of its analytical capacity. Amin focuses on what happens when the underlying asset follows ~~the~~ Merton's jump-diffusion process. He assumes that the stock price can move up or down by one tick in each discrete period, as was postulated by ~~as the~~ Cox, Ross, and Rubinstein's (1979) model (~~in each discrete period,~~ ~~[~~where a tick is the minimum possible change in the stock price)~~]~~. However, the asset price can also change on account ~~for~~ of a rare event (jump). Therefore, he also permits the underlying asset price to change by multiple ticks in a single period. This multiple-tick jump is the discrete time counterpart of the continuous time jump component. Based on the risk-neutral valuation argument and the assumption that the jump risk is diversifiable, his model ~~is demonstrated to be able to~~ weakly converges with ~~to~~ the theoretical option values under some mild regularity conditions.

Recently, Chang and Fu (2001) investigated the option pricing on traded assets, in settings where ~~when~~ either the underlying asset follows a jump diffusion process, or the volatility of the underlying asset is assumed to be stochastic. They extended the literature by combining the transformation technique of Hilliard and Schwartz (1996) and the discrete-time jump-diffusion model of Amin (1993), yielding ~~to develop~~ a bivariate binomial tree model. Furthermore, Kou (2002) offers a double exponential jump-diffusion model for the purpose of option pricing. He assumes the asset price follows a Brownian motion plus a compound Poisson process, with jump sizes registering as a doubled exponential increase in distribution~~being double exponential distributed~~. Under the assumption, the model can explain about the asymmetric leptokurtic feature and the volatility smile effect in options markets. Moreover, it can lead to analytic solutions to many option-pricing problems, including plain vanilla options, standard interest rate derivatives, and some path-dependent options.

Comment [PEH1]: CHECK: Unclear. Should it read something instead such as, "the working assumption of this model is that it can explain the asymmetric…etc"?

Among the stochastic volatility models, the GARCH~~-type of~~ model is one of the most comprehensive and popular ~~widely adopted~~ models used to capture smooth persistent changes in volatility. However, it is not able to explain~~ing~~ the large discrete changes embedded in asset returns. A ~~tractable~~ credible alternative is to incorporate a jump process into a GARCH-type model, and indeed, researchers have explored its possibilities. Jorion (1988) combined an ARCH model with a jump component to empirically examine foreign exchange rates and stock returns. ~~Similarly~~Likewise, Vlaar and Palm (1993), and ~~and~~ Nieuwland, Vershchoor, and Wolff (1994) adopted a constant jump intensity-GARCH model to capture foreign exchange rate dynamics. Furthermore, Lin and Yeh (2000) modify ~~the~~ Jorion's (1988) model to derive a new jump-diffusion-GARCH model, and provide empirical tests on the Taiwan stock market to examine whether discontinuous time paths exist or not. In their empirical studies, both Jorion (1988) and ~~and~~ Lin and Yeh (2000)~~, all~~ found that the combined models could provide a better explanation for the behavior of financial asset prices. Recently, Duan, Ritchken, and Sun (2004) tested the ~~GARCH-Jump~~GARCH-jump model using the S&P 500 data as a research sample. Their research ~~and~~ shows that the inclusion of jumps significantly improves the fit of historical time series ~~of~~ for the S&P 500 data. In option pricing, Duan, Ritchken, and Sun (2006) propose a new GARCH-jump model which takes the correlated systematic jump into account and solves option prices by simulation approaches. However, to the best of our knowledge, there is currently no ~~a~~ model available that can value ~~American style~~American-style

Comment [PEH2]: CHECK: this is the second time I believe this term has appeared. Do "large" and "discrete" need to be distinguished in this context, as logically something that is large is less discrete than a smaller entity? Perhaps use of "and/or" as a linkage point might clarify the intended meaning?

Comment [PEH3]: CHECK: They were able to solve option prices by simulation approaches because they had already taken the correlated systematic jump into account? If not, this sentence would be better served by breaking down into separate, clear components.

options. Such a model would need to be capable of capturing the dependency of those options ~~that depend~~ on an underlying asset, with respect to both a ~~under a~~ GARCH process ~~with~~ and a jump component ~~included is still lacking~~.

The purpose of this paper is to develop a discrete-time option pricing model which allows the underlying stock prices to follow a mixture of GARCH process and jump process. To this end we combine the lattice algorithms of ~~GARCH model of~~ Ritchken and Trevor's (1999) GARCH model and the discrete-time tree of jump-diffusion model of Amin (1993). ~~to derive a~~An integrated GARCH-jump option pricing model is thus attained~~.~~, ~~The new model~~ which provides ~~us~~ an efficient discrete-time lattice framework to price~~—,~~ in particular, ~~American style~~American-style options. It is also demonstrated that the model can provide ~~more~~ greater degrees of freedom to explain the skew feature of the stock returns distribution and capture the volatility smile and term structure of volatility effects on the options market. At the same time, the new GARCH-jump model, which contain~~ings~~s several nested models as degenerated cases, provides us with an efficient tool to conduct empirical tests on options pricing.

The remain~~ing~~der of this paper is organized as follows: i~~I~~n Section II, we construct a generalized lattice model under the GARCH-jump process and discuss its adaption and degeneration to the nested models. In Section III, we derive option pricing procedures ~~under~~ from the GARCH-jump lattice model. In Section IV, we ~~proceed~~ produce numerical an~~s~~alyses for the GARCH-jump lattice model in pricing options, and then consider its convergence behavior towards the nested models. Section V is the conclusion of this paper.

II. THE LATTICE MODEL WITH GARCH AND JUMPS

1. General Framework

Suppose that the price of the stock at time t, under the risk-neutral measure Q, with the time increment Δt, follows the following generalized GARCH-jump process:

$$\ln\left(\frac{S_{t+\Delta t}}{S_t}\right) = m_t \Delta t + \sqrt{h_t \Delta t}\,\mathbf{X}_t \,, \qquad (1)$$

where

$$m_t = m_t\left(h_t, \Delta t\right) = r_f - \frac{h_t}{2} - \lambda(\Delta t)[K_t - 1]\,,$$

$$\mathbf{X}_t = Z_t + \sum_{j=1}^{N_t^Q(\Delta t)} J_t^{(j)}\,,$$

$$Z_t \sim N^Q(0,1)\,,$$

$$J_t^{(j)} = J_t^{(j)}\left(h_t, \Delta t\right) \sim N^Q(\mu_J(h_t, \Delta t), \sigma_J^2(h_t, \Delta t))\,,$$

$K_t = K_t\left(h_t, \Delta t\right) = E\left(\exp\left(\sqrt{h_t \Delta t}\, J_t^{(j)}\right)\right)$, and

$N_t^Q\left(\Delta t\right)$ is a Poisson process with the jump intensity of $\lambda\left(\Delta t\right)\Delta t$ under the risk-neutral measure Q.

In the above specifications, r_f is the risk-free interest rate, and m_t and h_t denote the drift rate and the variance of the stock price process at time t. $\mathbf{X}_t$ is a compound Poisson normal process, which is a mixture of a standard normal process Z_t and a Poisson jump process J_t, with the jump intensity of $\lambda(\Delta t)$, and the jump magnitude following a normal distribution, whose mean and standard deviation, $\mu_J (h_t,\Delta t)$ and $\sigma_J (h_t,\Delta t)$, are assumed to be generally dependent on h_t and Δt. K_t denotes the average rate of jump plus 1.

The variance process of the stock price returns is assumed to follow the generalized process:

$$h_{t+\Delta t} - h_t = f\left(\nu_{t+\Delta t}, h_t, \Delta t\right)\,, \qquad (2)$$

where

$$\nu_{t+\Delta t} = \frac{(\ln S_{t+\Delta t} - \ln S_t - m_t \Delta t)/\sqrt{h_t \Delta t} - E^Q(\mathbf{X}_t)}{\sqrt{Var^Q(\mathbf{X}_t)}}\,,$$

is the standardized innovation of the stock price return process and

$$E^Q(\mathbf{X}_t) = \lambda(\Delta t)\Delta t \mu_J(h_t, \Delta t),$$

$$Var^Q(\mathbf{X}_t) = 1 + \lambda(\Delta t)\Delta t(\mu_J^2(h_t, \Delta t) + \sigma_J^2(h_t, \Delta t)),$$

~~are the mean~~is the mean and variance of the compound Poisson normal process, $\mathbf{X}_t$ ~~,~~; under the risk-neutral measure Q.

If the NGARCH process is considered, the variance process becomes:

$$f\left(v_{t+\Delta t}, h_t, \Delta t\right) = \beta_0 \Delta t + (\beta_1 - 1)h_t \Delta t + \beta_2(h_t, \Delta t)h_t \Delta t(v_{t+\Delta t} - c^Q(h_t, \Delta t))^2 . \quad (3)$$

The variance structure imposed in Equation (3) is a more general nonlinear asymmetric GARCH (NGARCH) model than those used by Engle and Ng (1993) and Duan (1995). The ~~nonnegative~~non-negative parameter $c^Q(h_t, \Delta t)$ signifies a negative correlation between the innovations of the stock price return and its conditional volatility under the risk-neutral measure Q. To follow the NGARCH process, the parameters have some restrictions with a typical GARCH process, which include $\beta_0 > 0$, $\beta_1 \geq 0$, and $\beta_2(h_t, \Delta t) \geq 0$ to ensure the positive conditional volatility.

Based on ~~the~~ Amin (1993) and Ritchken and Trevor's (1999) setting, given $y_t = \ln(S_t)$, the logarithmic stock price process of $y_{t+\Delta t}$, with an increment of time Δt, can be approximated in a lattice space as

$$y_{t+\Delta t} = y_t + j\gamma_n ; \quad j = 0, \pm 1, \pm 2, \ldots , \quad (4)$$

where γ_n is the size of changes in the stock price return to be defined later, and j denotes the index of the number of possible ticks changed in the stock price return for the GARCH-jump process.

In the lattice model, we assume that the stock price changes can be driven by a local component and a jump component, where the "local" means the variation of the stock price follows the assumption of a diffusion process, and the "jump" means that the stock price can change to an arbitrary level, either within or beyond the local change levels. According to the assumption in Amin (1993), the jump risk is diversifiable and is not priced in the market, and the local and jump changes are mutually exclusive[1]. ~~In o~~Our model makes allowance for the simultaneous occurrence of both kinds of price fluctuation~~, we further permit both of the two types of price changes can occur simultaneously~~.

Comment [PEH4]: CHECK: why? Because it is not priced in the market? Is it the frequency of risk that is diverse or the kinds of risks that are themselves diverse?

[1] Amin (1993) permits the two price changes ~~are~~ to be mutually exclusive for the expositional convenience. In the continuous time limit, it is irrelevant whether they are mutually exclusive or not.

Table 7. ~~Early Exercise~~Early-exercise Premium for Duan, Ritchken, and Sun's (2006) Model

This table shows the ~~early exercise~~early-exercise premium for Duan, Ritchken, and Sun's (2006) model. Parameters for the example are the same as in Table 5, and the number of variances for each node is M=50. Assume further that the risk-free interest rate $r_f = 0.05$, the initial stock price $S_0 = 500$, the strike price $X = 500$, and the time increment Δt is~~are~~ set to one day. We calculate ~~put option~~put-option prices for both European (Panel A) and American (Panel B) styles. As ~~we~~ can be seen in Panel C of the table, the ~~early exercise~~early-exercise premiums are generally small for the parameters used.

Panel A: European ~~Put Option~~Put-option Value

Strike Price	Time to Maturity						
	10	20	30	40	50	75	100
400	2.9765	6.0054	9.1632	12.5157	16.0488	25.3477	34.7745
450	4.4246	9.9807	16.0585	22.2045	28.2407	42.4718	55.3334
500	17.2640	28.0658	37.3759	45.7430	53.4028	70.2361	84.5885
550	58.7961	67.5285	75.7907	83.5027	90.6982	106.7569	120.5938
600	107.4635	114.5464	121.3153	127.7803	133.9406	148.0681	160.5761

Panel B: American ~~Put Option~~Put-option Value

Strike Price	Time to Maturity						
	10	20	30	40	50	75	100
400	2.9769	6.0068	9.1660	12.5204	16.0560	25.3634	34.8031
450	4.4252	9.9829	16.0631	22.2123	28.2527	42.4982	55.3808
500	17.2650	28.0691	37.3829	45.7551	53.4216	70.2774	84.6615
550	58.7974	67.5333	75.8010	83.5206	90.7259	106.8178	120.7003
600	107.4652	114.5531	121.3299	127.8056	133.9797	148.1534	160.7249

Panel C: ~~Early exercise~~Early-exercise ratio =(American Put—European Put)/American Put

Strike Price	Time to Maturity						
	10	20	30	40	50	75	100
400	0.01%	0.02%	0.03%	0.04%	0.05%	0.06%	0.08%
450	0.01%	0.02%	0.03%	0.04%	0.04%	0.06%	0.09%
500	0.01%	0.01%	0.02%	0.03%	0.04%	0.06%	0.09%
550	0.00%	0.01%	0.01%	0.02%	0.03%	0.06%	0.09%
600	0.00%	0.01%	0.01%	0.02%	0.03%	0.06%	0.09%

Table 8. Sensitivity Analysis for ~~Early Exercise~~Early-exercise Premium

This table shows the sensitivity analysis for option ~~early exercise~~early-exercise premium with the GARCH-jump model, with respect to jump intensity and magnitude parameters. According to Amin (1993), significant jumps ~~obstacle~~ impede the early exercise of American options. This reduces the ~~early exercise~~early-exercise premium and produces insignificant differences between American options and European options. As shown in this table, we ~~can~~ find the proportion of ~~early exercise~~early-exercise premium increases with the decreases in the jump intensity. Similarly, the proportion of ~~early exercise~~early-exercise premium increases with the decreases in the jump magnitude parameter. Parameters used are the same as those in Table 3, a~~399~~nd *M*=50. All numbers are in percent~~age~~ile value of the ~~early exercise~~early-exercise value relative to the American option value.

Panel A: Time to Maturity *T*=10 days

Mean of Jump magnitude μ_J	Jump intensity λ										
	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
-1	2.015	1.323	0.725	0.261	0.162	0.164	0.166	0.168	0.169	0.171	0.173
-0.75	2.015	1.269	0.629	0.184	0.141	0.143	0.143	0.145	0.144	0.145	0.147
-0.5	2.015	1.212	0.526	0.137	0.122	0.123	0.122	0.123	0.122	0.122	0.123
-0.25	2.015	1.151	0.419	0.109	0.105	0.105	0.104	0.103	0.102	0.102	0.102
0	2.015	1.086	0.310	0.090	0.090	0.089	0.087	0.086	0.084	0.084	0.083
0.25	2.015	1.016	0.201	0.076	0.076	0.074	0.072	0.071	0.069	0.068	0.067
0.5	2.015	0.947	0.124	0.065	0.063	0.061	0.059	0.057	0.055	0.054	0.054
0.75	2.015	0.875	0.078	0.054	0.052	0.050	0.048	0.046	0.044	0.043	0.042
1	2.015	0.800	0.054	0.045	0.043	0.040	0.038	0.036	0.035	0.034	0.033

Panel B: Time to Maturity *T*=50 days

Mean of Jump magnitude μ_J	Jump intensity λ										
	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
-1	5.193	3.825	2.397	1.609	1.260	1.144	1.089	1.045	1.006	0.969	0.935
-0.75	5.193	3.717	2.221	1.412	1.104	1.010	0.961	0.921	0.885	0.852	0.821
-0.5	5.193	3.597	2.029	1.216	0.960	0.884	0.840	0.803	0.770	0.740	0.713
-0.25	5.193	3.465	1.822	1.033	0.829	0.767	0.727	0.693	0.663	0.636	0.611
0	5.193	3.320	1.603	0.868	0.712	0.659	0.622	0.591	0.564	0.540	0.518
0.25	5.193	3.164	1.376	0.725	0.606	0.560	0.527	0.499	0.474	0.453	0.434
0.5	5.193	2.997	1.152	0.605	0.513	0.472	0.442	0.416	0.394	0.375	0.359
0.75	5.193	2.818	0.940	0.502	0.430	0.394	0.366	0.344	0.324	0.308	0.294
1	5.193	2.629	0.750	0.416	0.358	0.325	0.301	0.280	0.264	0.250	0.238